ECL 4340

POWER SYSTEMS

LECTURE 19
ECONOMIC DISPATCH

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ANNOUNCEMENTS

- Be reading Chapter 6, sections 6.12 & 6.13.
- HW 9 is due November 11, Friday.
- HW 10 is posted, due November 18, Friday.

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GENERATION DISPATCH

- Since the load is variable and there must be enough generation to meet the load, almost always there is more generation capacity available than load
- Optimally determining which generators to use can be a complicated task due to many different constraints:
  - For generators with low or no cost fuel (e.g., wind and solar PV) it is "use it or lose it"
  - For others like hydro there may be limited energy for the year
  - Some fossil has shut down and start times of many hours
- Economic dispatch looks at the best way to instantaneously dispatch the generation

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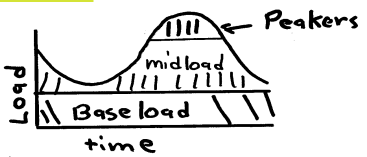
Generator Types

- Traditionally utilities have had three broad groups of generators
  - baseload units: large coal/nuclear; always on at max.
  - midload units: smaller coal that cycle on/off daily
  - peaker units: combustion turbines used only for several hours during periods of high demand



Wind and solar PV can be quite variable; usually they are operated at max. available power

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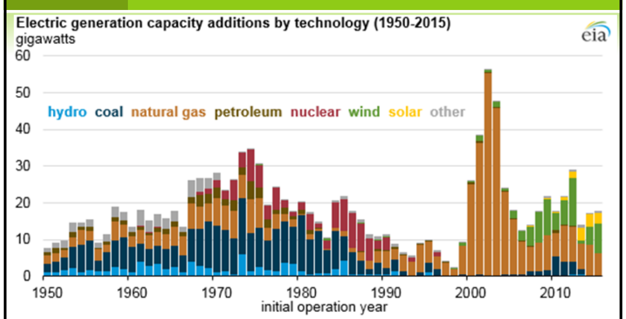
Utility-Scale Generator Capacity Additions in the US



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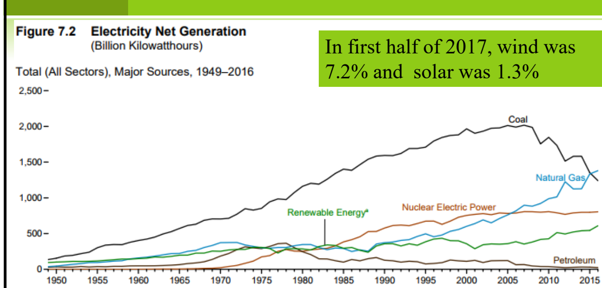
US Electricity Generation by Fuel Type



In first half of 2017, wind was 7.2% and solar was 1.3%

Source: www.eia.gov/totalenergy/data/monthly/pdf/sec7_4.pdf

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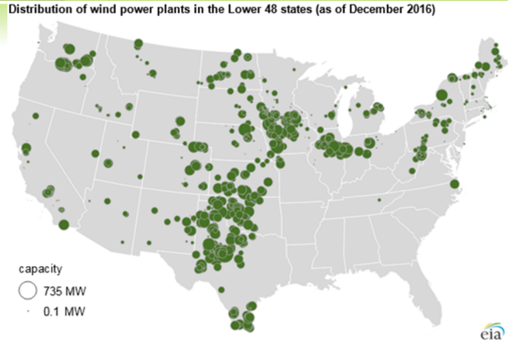
Distribution of Wind Power Plants

Distribution of wind power plants in the Lower 48 states (as of December 2016)



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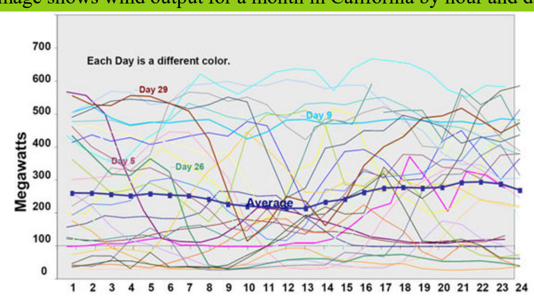
The Challenge of Wind Dispatch

Image shows wind output for a month in California by hour and day



Source: www.megawattsf.com/gridstorage/gridstorage.htm

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Thermal versus Hydro Generation

- The two main types of generating units are thermal and hydro, with wind rapidly growing
- For hydro the fuel (water) is free, but there may be many constraints on operation:
  - fixed amounts of water available
  - reservoir levels must be managed and coordinated
  - downstream flow rates for fish and navigation
- Hydro optimization is typically longer term (many months or years)

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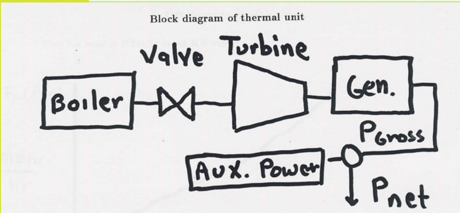
Block Diagram of Thermal Unit



To optimize generation costs we need to develop cost relationships between net power out and operating costs. Between 2-6% of power is used within the generating plant; this is known as the auxiliary power. 10

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Modern Coal Plant

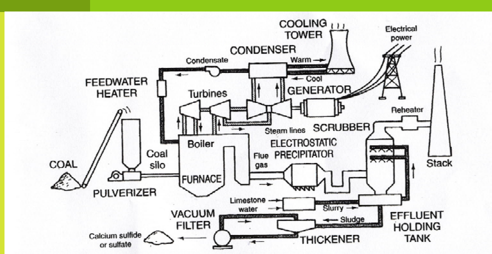

Figure 3.19 Typical modern coal-fired power plant using an electrostatic precipitator for particulate control and a limestone-based SO_2 scrubber. A cooling tower is shown for thermal pollution control. From Masters (1998).

Source: Masters, Renewable and Efficient Electric Power Systems, 2004 11

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Turbine for Nuclear Power Plant

Source: http://images.pennnet.com/articles/pe/cap/cap_gephoto.jpg 12

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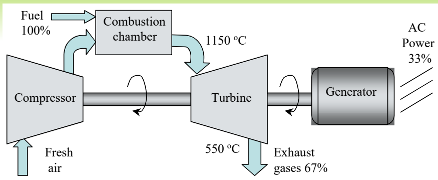
Basic Gas Turbine



Most common fuel is natural gas

Typical efficiency is around 30 to 35%

Maximum Efficiency

$$= 1 - \frac{550 + 273}{1150 + 273} = 42\%$$

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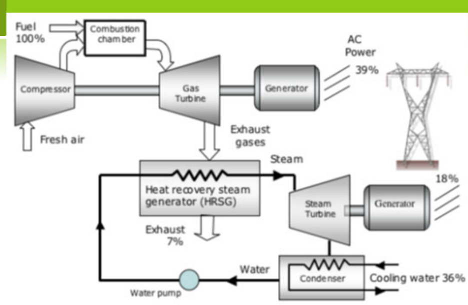
Combined Cycle Power Plant



Efficiencies of up to 60% can be achieved, with even higher values when the steam is used for heating. Fuel is usually natural gas. 14

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Generator Cost Curves

- Generator costs are typically represented by up to four different curves
  - input/output (I/O) curve
  - fuel-cost curve
  - heat-rate curve
  - incremental cost curve
- For reference
  - 1 Btu (British thermal unit) = 1054 J
  - 1 MBtu = 1x10^6 Btu
  - 1 MBtu = 0.293 MWh
  - 3.41 Mbtu = 1 MWh

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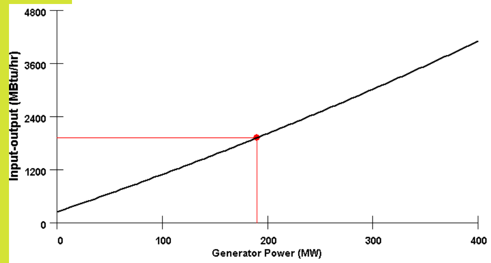
I/O Curve

- The IO curve plots fuel input (in MBtu/hr) versus net MW output.



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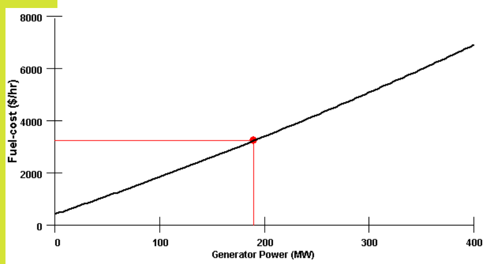
Fuel-cost Curve

- The fuel-cost curve is the I/O curve scaled by fuel cost. A typical cost for coal is $ 1.70/Mbtu.



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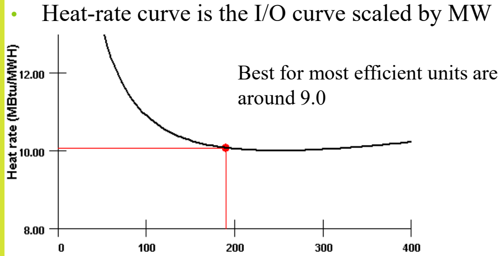
Heat-rate Curve

- Plots the average number of MBtu/hr of fuel input needed per MW of output.
- Heat-rate curve is the I/O curve scaled by MW



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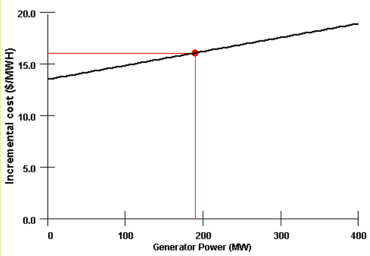
Incremental (Marginal) cost Curve

- Plots the incremental \$/MWh as a function of MW.
- Found by differentiating the cost curve



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Mathematical Formulation of Costs

- Generator cost curves are usually not smooth. However the curves can usually be adequately approximated using piece-wise smooth, functions.
- Two representations predominate:
  - quadratic or cubic functions
  - piecewise linear functions
- We'll assume a quadratic presentation:

$$C_i(P_{Gi}) \;=\; \alpha_i + \beta P_{Gi} + \gamma P_{Gi}^2 \quad \text{\$/hr (fuel-cost)}$$

$$IC_i(P_{Gi}) \;=\; \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \quad \text{\$/MWh}$$

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Coal Usage Example 1

- A 500 MW (net) generator is 35% efficient. It is being supplied with Western grade coal, which costs \$1.70 per MBtu and has 9000 Btu per pound. What is the coal usage in lbs/hr? What is the cost?

At 35% efficiency, the required fuel input per hour is

$$\frac{500\text{ MWh}}{\text{hr}\times 0.35} = \frac{1428\text{ MWh}}{\text{hr}} \times \frac{1\text{ MBtu}}{0.29\text{ MWh}} = \frac{4924\text{ MBtu}}{\text{hr}}$$

$$\frac{4924\text{ MBtu}}{\text{hr}} \times \frac{1\text{ lb}}{0.009\text{MBtu}} = \frac{547{,}111\text{ lbs}}{\text{hr}}$$

$$\text{Cost} = \frac{4924\text{ MBtu}}{\text{hr}} \times \frac{\$1.70}{\text{MBtu}} = 8{,}370.8\text{ \$/hr or \$16.74/MWh}$$

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Coal Usage Example 2

- Assume a 100W lamp is left on by mistake for 8 hours, and that the electricity is supplied by the previous coal plant and that transmission and distribution losses are 20%. How coal has been used?

With 20% losses, a 100W load on for 8 hrs requires 1 kWh of energy. With 35% gen. efficiency this requires

$$\frac{1\text{ kWh}}{0.35} \times \frac{1\text{ MWh}}{1000\text{ kWh}} \times \frac{1\text{ MBtu}}{0.29\text{ MWh}} \times \frac{1\text{ lb}}{0.009\text{MBtu}} = 1.09\text{ lb}$$

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Incremental Cost Example

For a two generator system assume

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

Then

$$IC_1(P_{G1}) = \frac{dC_1(P_{G1})}{dP_{G1}} = 20 + 0.02P_{G1} \quad \text{\$/MWh}$$

$$IC_2(P_{G2}) = \frac{dC_2(P_{G2})}{dP_{G2}} = 15 + 0.06P_{G2} \quad \text{\$/MWh}$$

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Incremental Cost Example, cont'd

If $P_{G1} = 250$ MW and $P_{G2} = 150$ MW Then

$$C_1(250) = 1000 + 20 \times 250 + 0.01 \times 250^2 = \$\ 6625/\text{hr}$$

$$C_2(150) = 400 + 15 \times 150 + 0.03 \times 150^2 = \$6025/\text{hr}$$

Then

$$IC_1(250) = 20 + 0.02 \times 250 = \$\ 25/\text{MWh}$$

$$IC_2(150) = 15 + 0.06 \times 150 = \$\ 24/\text{MWh}$$

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Economic Dispatch: Formulation

- The goal of economic dispatch is to determine the generation dispatch that minimizes the instantaneous operating cost, subject to the constraint that total generation = total load + losses

Minimize $\quad C_T \triangleq \sum_{i=1}^{m} C_i(P_{Gi})$

Such that

$$\sum_{i=1}^{m} P_{Gi} = P_D + P_{Losses}$$

Initially we'll ignore generator limits and the losses

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Unconstrained Minimization

- This is a minimization problem with a single equality constraint
- For an unconstrained minimization a necessary (but not sufficient) condition for a minimum is the gradient of the function must be zero, $\nabla \mathbf{f}(\mathbf{x}) = \mathbf{0}$
- The gradient generalizes the first derivative for multi-variable problems:

$$\nabla \mathbf{f}(\mathbf{x}) \triangleq \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1}, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_2}, \ldots, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_n} \right]$$

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Minimization with Equality Constraint

- When the minimization is constrained with an equality constraint, we can solve the problem using the method of *Lagrange Multipliers*
- Key idea is to modify a constrained minimization problem to be an unconstrained problem

That is, for the general problem

minimize $\mathbf{f}(\mathbf{x})$ $\quad$ s.t. $\mathbf{g}(\mathbf{x}) = \mathbf{0}$

We define the Lagrangian $\quad L(\mathbf{x},\boldsymbol{\lambda}) = \mathbf{f}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x})$

Then a necessary condition for a minimum is the

$\nabla L_{\mathbf{x}}(\mathbf{x},\boldsymbol{\lambda}) = 0 \quad$ and $\quad \nabla L_{\boldsymbol{\lambda}}(\mathbf{x},\boldsymbol{\lambda}) = 0$

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Economic Dispatch Lagrangian

For the economic dispatch we have a minimization constrained with a single equality constraint

$$L(\mathbf{P}_G,\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D - \sum_{i=1}^{m} P_{Gi}\right) \quad \text{(no losses)}$$

The necessary conditions for a minimum are

$$\frac{\partial L(\mathbf{P}_G,\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \quad (\text{for } i = 1 \text{ to } m)$$

$$P_D - \sum_{i=1}^{m} P_{Gi} = 0$$

Equal lambda criteria

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Economic Dispatch Example

What is economic dispatch for a two generator system: $P_D = P_{G1} + P_{G2} = 500$ MW and

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

Using the Largrange multiplier method we know

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

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Economic Dispatch Example, cont'd

We therefore need to solve three linear equations

$$20 + 0.02P_{G1} - \lambda = 0$$

$$15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

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Lambda-Iteration Solution Method

- The direct solution only works well if the incremental cost curves are linear and no generators are at their limits
- A more general method is known as the Lambda-iteration
  - the method requires that there be a unique mapping between a value of Lambda and each generator's MW output
  - the method then starts with values of Lambda below and above the optimal value, and then iteratively brackets the optimal value

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Lambda-Iteration Algorithm

Pick λ^{L} and λ^{H} such that

$$\sum_{i=1}^{m} P_{Gi}(\lambda^{L}) - P_D < 0 \qquad \sum_{i=1}^{m} P_{Gi}(\lambda^{H}) - P_D > 0$$

While $\left|\lambda^{H} - \lambda^{L}\right| > \varepsilon$, Do

$\lambda^{M} = (\lambda^{H} + \lambda^{L})/2$

If $\sum_{i=1}^{m} P_{Gi}(\lambda^{M}) - P_D > 0$, Then $\lambda^{H} = \lambda^{M}$

Else $\lambda^{L} = \lambda^{M}$

End While

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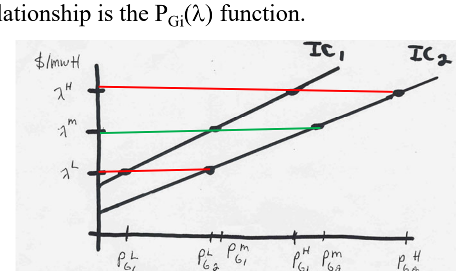
Lambda-Iteration: Graphical View

In the graph shown below for each value of Lambda, there is a unique P_{Gi} for each generator. This relationship is the $P_{Gi}(\lambda)$ function.



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Lambda-Iteration Example

Consider a three generator system with

$$IC_1(P_{G1}) = 15 + 0.02 P_{G1} = \lambda \quad \$/\text{MWh}$$
$$IC_2(P_{G2}) = 20 + 0.01 P_{G2} = \lambda \quad \$/\text{MWh}$$
$$IC_3(P_{G3}) = 18 + 0.025 P_{G3} = \lambda \quad \$/\text{MWh}$$

and with constraint $P_{G1} + P_{G2} + P_{G3} = 1000\,\text{MW}$

Rewriting as a function of λ, $\text{P}_{\text{Gi}}(\lambda)$, we have

$$\text{P}_{\text{G1}}(\lambda) = \frac{\lambda - 15}{0.02} \qquad \text{P}_{\text{G2}}(\lambda) = \frac{\lambda - 20}{0.01}$$
$$\text{P}_{\text{G3}}(\lambda) = \frac{\lambda - 18}{0.025}$$

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Lambda-Iteration Example, cont'd

Pick λ^L & λ^{H} so $\sum_{\text{i}=1}^{\text{m}} \text{P}_{\text{Gi}}(\lambda^L) - 1000 < 0$ and

$$\sum_{\text{i}=1}^{\text{m}} \text{P}_{\text{Gi}}(\lambda^H) - 1000 > 0$$

Try $\lambda^L = 20$, then $\sum_{i=1}^{m} P_{Gi}(20) - 1000 =$

$$\frac{\lambda - 15}{0.02} + \frac{\lambda - 20}{0.01} + \frac{\lambda - 18}{0.025} - 1000 = -670\text{ MW} \quad < 0$$

Try $\lambda^{\text{H}} = 30$, then $\sum_{i=1}^{m} P_{Gi}(30) - 1000 = 1230\text{ MW} \quad > 0$

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Lambda-Iteration Example, cont'd

Pick convergence tolerance $\varepsilon = 0.05$ \$/MWh

Then, iterate since $\left|\lambda^H - \lambda^L\right| > 0.05$

$$\lambda^M = (\lambda^H + \lambda^L)/2 = 25$$

Then since $\sum_{i=1}^{m} P_{Gi}(25) - 1000 = 280 > 0$, we set $\lambda^H = 25$

Since $|25 - 20| > 0.05$

$$\lambda^M = (25 + 20)/2 = 22.5$$

$$\sum_{i=1}^{m} P_{Gi}(22.5) - 1000 = -195 < 0, \text{ we set } \lambda^L = 22.5$$

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Lambda-Iteration Example, cont'd

Continue iterating until $\left|\lambda^{H} - \lambda^{L}\right| < 0.05$

The solution value of λ, λ^*, is 23.53 \$/MWh

Once λ^* is known we can calculate the P_{Gi}

$$P_{G1}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$P_{G2}(23.5) = \frac{23.53 - 20}{0.01} = 353 \text{ MW}$$

$$P_{G3}(23.5) = \frac{23.53 - 18}{0.025} = 221 \text{ MW}$$

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Generator MW Limits

- Generators have limits on the minimum and maximum amount of power they can produce
- Often times, the minimum limit is not zero. This represents a limit on the generator's operation with the desired fuel type
- Because of varying system economics usually many generators in a system are operated at their maximum MW limits.

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Lambda-Iteration with Gen Limits

In the Lambda-iteration method, the limits are taken into account when calculating $P_{Gi}(\lambda)$:

if $P_{Gi}(\lambda) > P_{Gi,\max}$ then $P_{Gi}(\lambda) = P_{Gi,\max}$

if $P_{Gi}(\lambda) < P_{Gi,\min}$ then $P_{Gi}(\lambda) = P_{Gi,\min}$

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